For the following problems, let $\Sigma = \{c, s\}$.

1. Prove the following language is not regular.

   $B_1 = \{w | w \text{ contains an equal number of } c's \text{ and } s's\}$

   - Assume $B_1$ is regular. This means the pumping lemma for regular languages holds – that is, any string $s$ of length at least $p$ that is in $B_1$ can be “pumped”.

   - Consider $s = \ldots$. Note $s \in B_1$ and $s$ has length at least $p$, so $s$ must be able to be “pumped”.

   - Given that the part we are pumping ($y$) must be non-empty, and that it must occur in the first $p$ symbols of $s$, we know that $y$ must:

     - Consider the string $s' \text{ which is created from “pumping” } y$ ______ times.

     \[ s' = \ldots \]

   - Note that $s' \notin B_1$, as our inferences about $y$ we made above mean:

     - We therefore have a contradiction – $B_1$ is regular so $s$ must be “pumpable”, but we have shown it is not “pumpable”. We reached this contradiction by assuming $B_1$ was regular. Therefore, $B_1$ is not regular.

2. Prove the following language is not regular.

   $B_2 = \{c^n s^m | n \leq m\}$

   - Consider $s = \ldots$.

   - We know that $y$ must: \ldots

   - $s' = \ldots$

   - $s' \notin B_2$ because:
3. Prove the following language is not regular.

\[ B_3 = \{ w \mid \text{the length of } w \text{ is a power of 2} \} \]

- Consider \( s = \)__________________________.
- We know that \( y \) must: __________________________
- \( s' = \)__________________________
- \( s' \notin B_3 \) because:

4. Prove the following language is not regular.

\[ B_4 = \{ wc^n \mid w \text{ is a string over } \Sigma \text{ of length } n \} \]

- Consider \( s = \)__________________________.
- We know that \( y \) must: __________________________
- \( s' = \)__________________________
- \( s' \notin B_4 \) because:

5. Prove the following language is not regular.

\[ B_5 = \{ w \mid w \text{ is a “balanced” string, with } c \text{ “opening” and } s \text{ “closing” } \} \]

(I’ve obviously made these terms up, but I’ll explain \( B_5 \) in class)

- Consider \( s = \)__________________________.
- We know that \( y \) must: __________________________
- \( s' = \)__________________________
- \( s' \notin B_5 \) because: